THE MUSIC OF THE SPHERES.

By Pliny Earle Chase.

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The cumulative evidences of a commensurable unity of law, in atmospheric tides, gaseous explosions, sonant and luminous vibration, molar and molecular inertia, satellite distances, solar and planetary rotation, gravitating and magnetic action, encourage a more minute investigation into the character of the planetary and stellar harmonies.

We are at present unable to introduce any considerations connected with the velocity of gravity, except the probability that it is uniform at all distances. It seems reasonable, however, to extend the oscillatory hypothesis to all primary forms of force, and to suppose that there may be some tendency to harmonic vibrations, of inconceivable velocity, which may have contributed both to the original arrangement and to the present stability of the solar system.

I have already shown (*ante*, xii, 518–522) a disposition to aggregation in accordance with the coördinates of logarithmic curves, not only within our immediate system, but also in the interstellar spaces. The same tendency is seen in the differences of atmospheric pressure at different altitudes, being a necessary consequence of gaseous or other analogous elasticity. I have also shown, in the foregoing communication on "Oscillatory Forces in the Solar System," that the alternate planets, commencing at either end of the series, are arranged in a more obvious geometrical progression than the intermediate planets, but that all appear to be posited by simple oscillatory laws.

A closer scrutiny has detected additional relations, involving not only the typical ratio between uniform luminous and gravitating motions (π), and the ratio of linear oscillation (3), but also, at the centre of inertia of the planetary belt, the ratio between the squares of the distances of comparable actual and virtual fall $\left(\frac{\pi^2}{8}\right)$.* The compared velocities being uniform, the distances vary as the times; but the distances fallen through by gravity, vary as the squares of the times, and, consequently, as the squares of the distances in uniform velocities. The difference between π and 3 is so slight, that there might, perhaps, be some room to question which of the two ratios is the more influential; but the following comparisons seem to show that the ratio of linear oscillation is, perhaps, slightly predominant:

* See foregoing communication on "Estimation of Solar Mass and Distance from the Equilibrium of Elastic and Gravitating Forces."

I. Alternate Planetary Series.

	Theoretic.	Observed.	Ratio.	Mus. Interval.*
Neptune	30.043	30.037	3.149	3.174
Saturn	9.563	9.539	3.130	3.174
Mean Asteroid	3.044	3.047	3.047	2.996
Earth, perihelion	.969	.966	3.137	3.174
Mercury, "	.308	.319		
Earth, mean	1.000	1.000		
Uranus	19.231	19.183	3.687	3.776
Jupiter	5.196	5.203	3.416	3.364
Mars	1.522	1.524	2.106	2.118
Venus	.711	.723		

The closeness of approximation between the theoretic and observed values, and the near accordance of the ratios to musical intervals, induced me to apply the test of a harmonic progression, based upon the centre of linear oscillation ($\frac{2}{3}$), the general expression for the several terms being $\frac{2}{3n}$. I found that the mean planetary positions could be indicated by terms of such a progression, even more precisely than by Bode's law, inasmuch as the asteroidal belt is skipped, and additional light is thrown upon planetary eccentricities.

We have therefore three coördinate agencies, one of centrifugal oscillation, represented by my modification of Bode's law; one of centripetal oscillation, represented by the above harmonic law; and one of uniform gravitating oscillation, combined with the square of the ratio of the time of fall to the time of revolution, represented by $\frac{\pi^n}{32}$. The two former series seem to be attributable to the combined centripetal and centrifugal activities, at the two principal centres of inertia in our system, Sun and Jupiter. I therefore take the mean perihelion distance of Jupiter as the *point d' appui* of my harmonic series, and construct the Bode series in such manner that its sum shall be equivalent to the corresponding harmonic sum. The importance of the $\frac{\pi^n}{32}$ series, is obvious at a glance. The doubling of the second term of that series, to give the bracketed mean distance of Uranus, and the halving of the third term, to give the bracketed mean distance of Mars, are curiously suggestive.

In the following table, m denotes mean distance; p, perihelion; a, aphelion. The musical intervals represent the number of semitones in the ideal scale, proportionate to the corresponding theoretic planetary distances from Jupiter's mean perihelion. The planetary eccentricities are subject to such variations, that it is obviously important, in a preliminary comparison of this kind, to consider the secular *mean* aphelia and perihelia.

*According to the ideal scale in which each semitone is $\sqrt[12]{2}$.

II. Triple Planetary Series.

		Observed.		Harmonic.	Bode.	$\pi^n \div 32$	Intervals.
Neptune,	m	30.0370				$\pi^6 \div 32 = 30.0435$	31
"	p	29.7353	$6 \times ♃ p$	29.8692	29.5303		30
Uranus,	a	20.0432	$\frac{2}{3}$ Ψ	19.9128	19.8218		24
"	m	19.1826				[$\pi^5 \div 16 = 19.1262$]	22
Saturn,	a	10.	$\frac{2}{6}$ "	9.9564	10.1131		12
"	m	9.5389				$\pi^5 \div 32 = 9.5631$	11
Jupiter,	m	5.2028			5.2588		1
"	p	4.9782	Base	4.9782			0
Asteroid = 2♂		3.0440			2.8317	$\pi^4 \div 32 = 3.0440$	$8\frac{1}{2}$
Mars,	a	1.6442	$\frac{2}{36}$ Ψ	1.6594	1.6181		18
"	m	1.5237				[$\pi^4 \div 64 = 1.5220$]	$20\frac{1}{2}$
Earth.	m	1.	$\frac{2}{60}$ "	.9956	1.0133		28
"	p	.9661				$\pi^3 \div 32 = .9689$	28
Venus,	m	.7233	$\frac{2}{84}$ "	.7112			34
"	p	.6978			.7079		
(♀ + ☿) ÷ 2		.5552	$\frac{2}{108}$ "	.5531			38
Mercury,	a	.4555	$\frac{2}{132}$ "	.4526			$41\frac{1}{2}$
"	m	.3871	$\frac{2}{156}$ "	.3829	.4045		$44\frac{1}{2}$
"	p	.3187	$\frac{2}{180}$ "	.3319		$\pi^2 \div 32 = .3084$	47, 48
Sum, Theoretic				69.8033	68.4658	64.5761	
" Observed				69.8407	68.7104	64.6110	

The significance of the above table may, perhaps, be increased, by the following considerations:

1. Each series *precisely* represents actual planetary positions.

2. The combined influence of the three series is such as to produce planetary eccentricity, and to give an approximate value of such eccentricity for each planet.

3. In centrifugal emanation and centripetal acceleration, the uniform velocity of radial light and circular revolution, orbital inertia and tendency to centres of linear oscillation, elasticity and harmonic undulations in elastic-media, there are *verae causae* which might lead us to look for each of the three series.

4. Each series seems to exert a nearly equal influence upon the planetotaxis, as is seen by the observed and theoretic sums.

5. The accordance between the theoretic and observed values, is almost precisely equal in the harmonic and circular series (within about $\frac{1}{19}$ of one per cent.) and more than six times as close as in the Bode series (more than $\frac{1}{3}$ of one per cent).

6. The two predominant gravitating tendencies in the solar system are towards the Sun and towards Jupiter.

7. The centre of gravity of the Sun and Jupiter is at about 1.066 × solar radius from the Sun's centre. The mean aphelion centre of planetary inertia (9.6615) is about 1.069 × the mean perihelion centre. Jupiter's mean eccentricity (.0432) is about $\frac{2}{3}$ × the solar-jovian eccentricity (.066).

8. The mean perihelion distance of Jupiter corresponds with the distance of a planetary revolution which would be synchronous with the mean sun-spot period. The mean perihelion of Saturn (9.078), corresponds with the mean perihelion centre of planetary inertia (9.039).

9. The eighth term of the $\frac{\pi^n}{32}$ series ($2.137 \times$ solar radius), is equivalent to about twice the distance of the solar-jovian centre of gravity (1.066). The third term of the same series is about twice the mean radius vector of Mars.

10. The Saturnian year is to the terrestrial year, nearly as the solar rotation is to terrestrial rotation, or as solar is to terrestrial superficial gravity.

11. The harmonic series between Jupiter and the inner limit of the planetary belt ($\frac{1}{1}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, $\frac{1}{9}$, $\frac{1}{11}$, $\frac{1}{13}$, $\frac{1}{15}$), is remarkably simple. If we interpolate as many harmonic means between each of the terms, as are equivalent to the number of remaining planets (7), the series may be extended to the outer planetary limit.

12. The ultra-jovian series (2, 4, 6, ♃; or $\frac{2}{6}$, $\frac{2}{3}$, $\frac{2}{2}$ Ψ) renders each of the supra-asteroidal planets subservient to the stability of the system. For if all the exterior planets were in conjunction, Jupiter would be near the centre of linear oscillation, between their centre of gravity and the Sun; when Jupiter and Saturn are in opposition, Saturn, moving more slowly than Jupiter, may be regarded as somewhat pivotal, and the Sun is near the centre of linear oscillation; the Sun is also near the centre of linear oscillation when Saturn and Uranus are in opposition; Uranus is near Neptune's centre of linear oscillation, when those two planets are in conjunction, and the time of luminous oscillation between Uranus and the Sun's surface, appears to be *precisely* coincident with the time of planetary oscillation at the Sun's surface.

13. The Bodeian, Circular, and Harmonic Series, all concur in indicating alternate planetary positions, except in the asteroidal belt, which is at the extremity of linear oscillation when Jupiter and the Sun are regarded as fixing the length of the simple pendulum.

14. The eccentricity indicated within the asteroidal belt [$(3.044 - 2.832) \div 2.832$], is nearly equivalent to the mean eccentricity of Mars (.079). The Bodeian asteroidal term represents twice the mean perihelion distance, while the circular term represents twice the mean distance, of Mars (2.8317, 2.8064; 3.044, 3.047), and, very nearly, the ratio of Jupiter's mean perihelion to Mars's mean distance (3.028).

15. The inner limit of the asteroidal belt is near the linear centre of oscillation of the outer limit.

16. The musical intervals are generally such as to produce chords between any two adjacent theoretical planetary positions. But where quarter-tones occur, the discordant vibrations seem to have broken up or disturbed the tendencies to planetary aggregation, aiding in producing the asteroidal belt, giving Mars and Mercury their great eccentricity, and obliterating the planet between Mercury and Venus.

17. Centres of linear oscillation are found within the asteroidal belt at the oppositions of Neptune and Uranus, Neptune and Saturn, Uranus and Jupiter. The immense inertia of the Sun seems, in each of these cases, to have disturbed, without wholly destroying, the tendencies to planetary aggregation.

18. A careful examination shows numerous other perihelion, mean, or aphelion distances, connected with centres of linear oscillation, some of which I propose to enumerate in a subsequent communication.

19. Saturn's mean aphelion (10) is a mean proportional between Jupiter's mean perihelion (4.978) and the mean aphelion of Uranus (20.043), as well as between Uranus's mean perihelion (18.322) and Jupiter's mean aphelion (5.427).

20. The geometric mean of the theoretical harmonic planetary distances (2.13716), varies less than one-fortieth of one per cent. from the geometric mean of the actual planetary distances (2.13665). This approximation corresponds in precision to the one given, in my paper on "The Gamuts of Sound and Light," between the mean wave length of the Fraunhofer lines and those of the musical Scale.

21. The inner limit of the asteroidal belt (Flora = 2.2) is nearly coincident with the geometric planetary mean.

22. The year at the harmonic theoretical aphelion of Mars ($1.659^{\frac{3}{2}}$ = 2.1376), is to the terrestrial year, as the geometric planetary mean is to the Earth's mean radius vector. By means of this ratio, Pierce's Phyllotactic series may, perhaps, be connected with the harmonic series.

23. The linear centre of oscillation of Jupiter's radius vector, and the shorter extremity of the solar-jovian linear pendulum of which the Sun occupies the centre of oscillation, are both in the asteroidal belt.

24. The potential of *vis viva* in a gravitating mass (mv^2) varies as the product of the mass by the distance (md, or at the surface, mr), therefore the product of the mass by the radius may, perhaps, represent the limiting excursion of communicated elastic undulation at the surface. Such is at least the case, if the theoretical velocities of sound and light are the limiting velocities at the Earth and Sun.

25. The velocity communicated, during a given angular rotation, by Earth and Jupiter (the controlling orbs of the intra and extra-asteroidal systems) to falling bodies at their surface, is to the velocity communicated by the Sun, in an equivalent angular rotation, as planetary velocity at the Sun's surface, is to the velocity of light.

26. The centres of linear oscillation between Neptune's mean aphelion and Uranus's mean perihelion (2.1019), and between Neptune's perihelion and Uranus's aphelion (3.4503), are near the apparent inner and outer limits of the asteroidal belt.

It gives me great pleasure to acknowledge my indebtedness to Stockwell's discussion of the secular variations of the planetary elements, for the amount of verification his results have afforded to my hypotheses. I subjoin his determinations of the mean eccentricities, and the mean perihelia and aphelia which represent those determinations.

	E	*p*	*a*		*E*	*p*	*a*
Mercury,	.1766064	.3187	.4555	Jupiter,	.0431601	4.9782	5.4274
Venus,	.0353165	.6978	.7489	Saturn,	.0483504	9.0777	10.
Earth,	.0338676	.9661	1.0339	Uranus,	.0448614	18.3221	20.0432
Mars,	.0790650	.14032	1.6442	Neptune,	.0100389	29.7353	30.3385

ON THE FLAT-CLAWED CARNIVORA OF THE EOCENE OF WYOMING.

By Edward D. Cope, A. M.

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MESONYX. Cope.

This genus was described by the writer in the Proceedings of the American Philosophical Society for 1872, p. 460. It was there referred to the *Carnivora*, and stated to resemble *Hyænodon* in some respects. I propose on the present occasion to attempt a more exact determination of its structure and relationships. The only species yet certainly referable to it is *Mesonyx obtusidens*, Cope, l. c., which is represented by a fragmentary skeleton. There are preserved, portions of the skull with the teeth, chiefly mandibular; numerous vertebræ from all parts of the column; parts of scapula, ulna and fore feet; portions of pelvis, femora, tibiæ, tarsals, metatarsals. and phalanges.

The numerous unguiculate digits, the sectorial character of the molar teeth and the characteristic form of the astragalus demonstrate this genus to belong to the *Carnivora fissipedia*. It becomes interesting then to determine the relations of an Eocene type of the order to the families now living.

The cervical *vertebræ* are damaged. The dorsals are strikingly smaller than the lumbars, being less than half their bulk. They are opisthocœlian with shallow cups, and the centra are quite concave laterally and inferiorly. The centra of the lumbars are more truncate, with a trace of the opisthocœlian structure, and are quite depressed in form. The median part of the series is more elongate than in the corresponding vertebræ of the genus *Canis*. They exhibit an obtuse median longitudinal angle, on each side of which, at a little distance, a nutritious artery entered by a foramen. The zygapophyses of the posterior lumbars